

Chapter 1 Review

Functions:

- 1) Find the linear function $f(x)$ if $f(3) = 5$ and $f(-1) = 7$. Write the equation for $f(x)$ in both point-slope and slope-intercept form.
- 2) Find the domain of each function algebraically and support your response graphically.
 - a) $f(x) = \sqrt{4 - 5x}$
 - b) $f(x) = \frac{x - 4}{x^2 + 5x - 36}$
- 3) If $f(x) = 3x^2 + 1$ and $g(x) = \sqrt{3x - 2}$, find the following and simplify:
 - a) Evaluate $f(-1)$ and $g(2) - 2$ and $f(x + 2)$
 - b) Find the domains of $f(x)$ and $g(x)$ algebraically and support graphically.
 - c) Determine whether $f(x)$ and $g(x)$ have an inverse function. If so, find $f^{-1}(x)$ and $g^{-1}(x)$.
 - d) Find $(f - g)(3)$ and $(f + g)(-2)$
 - e) Find $(f \circ g)(x)$ and $(g \circ f)(x)$, simplify, and find their domains algebraically.

Exponential and Logarithmic Functions

- 4) For each problem below, write the equation in both exponential and logarithmic form and solve for x algebraically. Give answers to three decimal places.

a) $\log_7 42 = x$

b) $e^x = 15$

c) $x^4 = 96$

- 5) Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. Assume all variables are positive.

$$\ln \frac{x^4 \sqrt{y}}{z^5}$$

- 6) In a group project in learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be

$$P = \frac{0.83}{1 + e^{-0.2n}}$$

- a) Graph the equation and determine any horizontal asymptotes of the problem. Interpret the meaning of the upper asymptote in the context of the problem.
- b) After how many trials will 60% of the responses be correct? Solve algebraically.

- 7) Solve each equation for x algebraically. Be careful of the domain of each function and extraneous solutions.

a) $2e^{5x} = 18$ b) $\log_3 x + \log_3(x - 8) = 2$ c) $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

- 8) You invest \$3,000 in an account that pays 6.5% interest compounded monthly.

- a) How much will your investment be worth in 16 years?
b) How long will it take your investment to double?
c) If the interest is compounded continuously, how much will it be worth in 16 years?

Inverse Functions

- 9) Find the inverse of each function below and tell whether the inverse is a function.

a) $4x + 3y = 12$

b) $x^2 - 3y = 5$

Regression:

- 10) An administrator at the University of Colorado wants to find a function that relates a student's college grade point average G to the high school grade point average x . She randomly selects eight students and obtains the following data:

High School GPA, x	College GPA, G
2.73	2.43
2.92	2.97
3.45	3.63
3.78	3.81
2.56	2.83
2.98	2.81
3.67	3.45
3.10	2.93

- a) Does the relation defined by the set of ordered pairs (x, G) represent a function? Explain.
b) Draw a scatter plot for the data.
c) Find a best-fit line relating high school GPA and college GPA. Write in function notation.
d) Give the real-world meaning of the slope.
e) What is the relevant domain for the problem?
f) Use your model to predict a student's high school GPA if their college GPA is 3.23.

Trigonometry:

11) Find all trigonometric function values for each angle.

a) $\cos \theta = -\frac{3}{5}$, in quadrant III

b) $\tan \theta = -\frac{15}{8}$, in quadrant II

12) Give the amplitude, period, vertical translation, phase shift, and the graph (without your calculator) for the function $y = \frac{1}{2} \csc(2x - \frac{\pi}{4})$.

13) Prove the following Identities.

a) $\frac{\sin^2 \alpha}{\cos^2 \alpha} + \sin \alpha \csc \alpha = \sec^2 \alpha$

b) $\sec \theta = \cos \theta + \tan \theta \sin \theta$

c) $\frac{\cos A}{\sec A} + \frac{\sin A}{\csc A} = \sec^2 A - \tan^2 A$

d) $\frac{\sec^2 \beta - 1}{\sec \beta - 1} = 1 + \frac{1}{\cos \beta}$

e) $\sin 2\alpha = \frac{2}{\sec \alpha \csc \alpha}$

14) Solve the equations over the interval $[0, 2\pi)$.

a) $2\sin(x) - 1 = 0$

b) $3\sin^2(x) - 5\sin(x) + 2 = 0$

Answers to Calculus Ch. 1 Review:

- 1) point-slope form: $y = -\frac{1}{2}(x - 3) + 5$ or $y = -\frac{1}{2}(x + 1) + 7$
Slope-intercept: $y = -\frac{1}{2}x + 6.5$
- 2) a) $x \leq 0.8$
b) $x \neq -9, 4$
- 3) a) $f(-1) = 4$ $g(2) - 2 = 0$ $f(x+2) = 3x^2 + 12x + 13$
b) $f(x)$ all reals $g(x)$ $x \geq 2/3$
c) $f(x)$ no inverse $g(x)$ has an inverse $= (x^2 + 2)/3$
d) $(f - g)(3) \approx 25.35$ $(f + g)(-2) = \text{no solution}$
e) $(f \circ g)(x) = 9x - 5$, domain $x \geq 2/3$ $(g \circ f)(x) = \sqrt{9x^2 + 1}$, domain all reals
- 4) a) $\log_7 42 = x$, $7^x = 42$, $x = 1.921$
b) $e^x = 15$, $\ln 15 = x$, $x = 2.708$
c) $x^4 = 96$, $\log_x 96 = 4$, $x = 3.130$
- 5) $4\ln(x) + \frac{1}{2}\ln(y) - 5\ln(z)$
- 6) a) Asymptotes at $y = 0$ and $y = 0.83$. The proportion will approach, but not exceed 83% as the number of trials increases.
b) After 5 trials
- 7) a) 0.439
b) $x = 9$ ($x = -1$ is extraneous)
c) No Solution, $x = -2, -3$, but neither are in the domain
- 8) a) \$8,463.86
b) ≈ 128.3 months or ≈ 10.7 years
c) \$8,487.65
- 9) a) $y = 3 - \frac{3}{4}x$ yes it is a function
b) $y = \pm\sqrt{(5 + 3x)}$ no it is not a function
- 10) a) This is a function because every input only has one output.
b) Use calculator to get scatter plot
c) $f(x) = 0.96x + 0.7$
d) slope: on average the college GPA increases by 0.96 for every increase of 1 in GPA in high school.
y-int: A high school GPA of 0 would correspond with a college GPA of 0.07.
Practically this makes more sense as just an adjustment for the line since the first meaning doesn't make much sense.
e) The domain should either be $[0, 4]$ or $[0, 5]$ depending on the GPA system used at the high school.
f) About 3.19

- 11) $\sin\theta = -4/5$; $\cos\theta = -3/5$; $\tan\theta = 4/3$; $\csc\theta = -5/4$ $\sec\theta = -5/3$; $\cot\theta = 3/4$
b) $\sin\theta = 15/17$; $\cos\theta = -8/17$; $\tan\theta = -15/8$; $\csc\theta = 17/15$ $\sec\theta = -17/8$; $\cot\theta = -8/15$
- 12) Amp. = Not Applicable, Per = π , Vert. Shift = 0, Horz. Shift = $\pi/8$ right,
Graph on your calculator to check.
- 13) Solutions will vary, but all should work
- 14) a) $\pi/6$ and $5\pi/6$
b) 0.730, $\pi/2$, 2.412